Pricing Survivor Bonds with Affine-Jump Diffusion Stochastic Mortality Models

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Pricing Survivor Bonds with AJD models

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Capital-market-based solutions are an interesting alternative to reinsurance-based options for managing systemic longevity risk in pension funds, insurance companies, and annuity providers. The pricing of longevity-linked securities depends both on the stochastic process for the underlying risk factors (age-specific mortality rates, interest rate) and on the investor's risk attitude. This paper proposes a pricing approach for survivor bonds using affine-jump diffusion stochastic mortality models. The model structure uses a non-mean reverting square-root jump-diffusion Feller process combined with a Poisson process with double asymmetric exponentially distributed jumps to account for both negative and positive jumps. The model offers analytical tractability, fits well data, and allows for closed-form expressions for the survival probability. Illustrative empirical results on the pricing of survivor bonds are provided using U. S. mortality data for representative cohorts. The results suggest the cost of hedging longevity risk by issuing survivor bonds would be acceptable for the issuer.

CCS CONCEPTS • Continuous mathematics • Law, social and behavioral sciences • Modeling and simulation

Keywords: Survivor bonds, stochastic mortality models, Affine-jump diffusion, longevity risk, longevity-linked securities

1 INTRODUCTION

Public and private pension schemes provide an ex-ante efficient risk pooling mechanism that addresses the (individual) uncertainty of death through the delivery of a lifetime annuity, redistributing income in a welfare-enhancing manner. Without such an instrument, individuals risk outliving their accumulated wealth or leaving unintended bequests to his/her dependents [1-2, 25, 40]. Pension funds, insurance companies, annuity providers, and life settlement investors face long-run solvency challenges to provide guaranteed lifetime income due to uncertain financial returns and systematic (non-diversifiable) longevity risk. This risk is amplified by the current problems in state-run social security and healthcare systems. For pension plans and annuity providers, traditional longevity risk management solutions include loss control techniques, e.g., via product re-design or risk-sharing arrangements between pensioners/policyholders and providers [3-4], natural hedging, liability selling via an insurance or reinsurance contract (pension buy-outs/ins, bulk annuity transfers) and Insurance-Based Longevity Swaps. Traditional reinsurance is not a definitive answer to the problem due to the undiversifiable nature of systematic longevity risk. In recent years, several capital-market-based solutions for mortality and longevity risk management have been proposed and, some, successfully launched. They include insurance securitization, mortality- or longevity-linked securities such as CAT mortality bonds, survivor/longevity bonds [5], and derivatives with both linear and nonlinear payoff structures, e.g., Index-based Capital-market longevity swaps [6-7], q-forwards [8], S-forwards, K-forwards [9], mortality options, survivor options [7], survivor swaptions [10], K-options [11] and call-spreads [12].

This paper develops a pricing approach for survivor bonds using affine-jump diffusion stochastic mortality models. Survivor bonds are debt instruments with coupon and or principal payments linked to the dynamics of a reference population longevity index. The longevity index provides information on the survival probability of a

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given cohort aged x at time 0. The pricing of survivor bonds depends both on the stochastic process for the underlying risk factors (age-specific mortality rates, interest rate) under a risk-neutral (equivalent-martingale) probability measure and on the investor's risk attitude. In this paper, we use a risk-neutral valuation approach to incorporate the market price of longevity risk.[1] In the actuarial, financial, and demographic literature, several single and multiple-population continuous-time stochastic mortality models have been proposed for modelling the dynamics of mortality rates (see, e.g., [7, 13-17] and references therein), along with several individual discrete-time extrapolative models (see, e.g., [18-21] and references therein) and, more recently, model combinations [22-24, 36-39, 44]. In this paper, we follow [7] and use a non-mean reverting square-root jump-diffusion Feller process combined with a Poisson process with double asymmetric exponentially distributed jumps [27] to account for both negative (e.g., medical breakthroughs) and positive jumps (e.g., pandemics) of different sizes. Previous research considering jump processes in stochastic mortality modelling focused almost exclusively on the impact of negative mortality jumps to describe the dynamics of longevity improvements. The model offers analytical tractability, fits well data, and allows for closed-form expressions for the survival probability, permitting efficient computation of survivor bond prices and risk measures. We provide illustrative empirical results on the pricing of survivor bonds (of different term) using U. S. mortality data for representative cohorts and analyze the sensitivity of computed prices to key model parameters. The remainder of the paper is organized as follows. Section 2 outlines the key concepts and research methods used in the paper. Section 3 reports and discusses the survivor bond pricing results. Section 4 concludes.

2 MATERIALS AND METHODS

2.1 Survivor bond design

Consider a default[2] risk-free longevity zero-coupon bond $Z_x(t,T)$ paying the realized proportion of the initial population in cohort x that is alive at time T. The price of a longevity zero-coupon bond can be expressed as

$$Z_x(t,T) = E^{\mathbb{Q}}\left[e^{-\int_t^T (r(s)+\mu_{x+s}(s))ds} | \mathcal{F}_t\right] = E^{\mathbb{Q}}\left[e^{-\int_t^T r(s)ds} | \mathcal{H}_t\right] E^{\mathbb{Q}}\left[e^{-\int_t^T \mu_{x+s}(s)ds} | \mathcal{M}_t\right] = P(t,T)\, S^{\mathbb{Q}}(x,t,T), \quad (1)$$

where $\{\mu_x: t \geq 0\}$ is the mortality intensity process, $\{r(s): t \geq 0\}$ is the risk-free instantaneous interest rate process, $P(t,T)$ is the price at time t of a zero-coupon risk-free bond maturing at time T associated to the equivalent forward measure, and $S^{\mathbb{Q}}(x,t,T)$ is the risk-neutral survival probability to time T of a cohort aged x at time t. Following [32], we consider a coupon-at-risk index-linked survivor bond design in which the floating coupon at time T is linked to the deviation of the actual survival probability $S^A(x,t,T)$ to the reference life table (best estimate $S^{BE}(x,t,T)$). Specifically, the general form of the coupon at time t is:

$$C_t = \theta\left(1 + S^{BE}(x,t,T) - S^A(x,t,T)\right) + \pi, \quad (2)$$

where θ is the bond's standard coupon and π is the bond's additive spread (margin) corresponding to the risk premium paid to the investor who will assume longevity risk (i.e., the risk that $S^A(x,t,T) > S^{BE}(x,t,T)$). From (2) the risk passed to the financial market is the risk that future survival probabilities exceed those estimated at contract initiation, a design structure that serves as a hedging instrument to the issuer and is likely to attract

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[1] Alternative approaches include the Wang transform, the instantaneous Sharpe ratio method, the Equivalent Utility Pricing Principle, the Cost of Capital approach, multivariate exponential tilting or the CAPM- (and CCAPM) based approaches.

[2] For a discussion of credit risk models see, e.g., [26, 28-31, 42,43] and references therein.

investors interested in portfolio diversification, being rewarded by the risk premium. Following [32], we derive the survivor's bond price using an indifference pricing approach as follows:

$$\sum_{t=1}^{M} \theta\, P(0,t) + P(0,M) = \sum_{t=1}^{M} \mathbb{Q}[C_t]\, P(0,t) + P(0,M), \tag{3}$$

where the left-hand side in (3) represents the fair value of a straight bond paying a fixed annual coupon θ whereas the right-hand side is the fair value of the coupon-at-risk survivor bond with $\mathbb{Q}[C_t]$ denoting the risk-neutral certainty equivalent to the future random cashflow C_t. Let $\mathbb{Q}$ be a risk-neutral longevity risk measure. Equation (3) becomes:

$$\pi^* = \frac{\pi}{\theta} = \frac{\sum_{t=1}^{M} P(0,t)[S^{A,\mathbb{Q}}(x,t,T) - S^{BE}(x,t,T)]}{\sum_{t=1}^{M} P(0,t)}, \tag{4}$$

where π^* is the relative additive margin of the survivor bond, i.e., the absolute risk margin as a proportion of an equivalent fixed coupon paying bond. Equation (4) shows that the more actual survival probabilities deviate from those estimated at contract initiation the higher the relative additive margin of the survivor bond will be to compensate the investor for taking longevity risk.

2.2 Affine-Jump Diffusion Stochastic Mortality Models

Let τ_x denote a non-negative random variable representing the residual lifetime of an individual aged x at present time $t = 0$. We consider the time interval $[0, \omega]$, with ω denoting the highest attainable age, and define the stochastic force of mortality process on a complete filtered probability space (Ω, G, P). The stopping time τ_x is said to admit an intensity μ_x if the compensator of the counting process does. Under this setting, the remaining lifetime of an individual is a doubly stochastic stopping time with intensity μ_x. Assume that, under the real world (or physical) probability measure P, the mortality intensity of an individual aged $x+t$ at time t, $\mu_{x+t}(t)$, is driven by a non-mean reverting square-root affine jump-diffusion process combined with a Poisson process with double asymmetric exponentially distributed jumps, i.e.,

$$d\mu_{x+t}(t) = a\mu_{x+t}(t)dt + \sigma\sqrt{\mu_{x+t}(t)}dW_t + d\Big(\sum_{i=1}^{N_t^P} Y_i^P\Big), \tag{5}$$

where $\mu_x(0) > 0$, $a, \sigma > 0$, W_t is a P-measured standard Brownian motion, and N_t^P is a P-measured standard Poisson process with intensity η. The jump sizes Y_i^P are i.i.d. random variables with the asymmetric double exponential density of [27]

$$f(y) = \frac{\delta_1}{\vartheta_1} e^{-\frac{y}{\vartheta_1}} \mathrm{I}_{\{y \ge 0\}} + \frac{\delta_2}{\vartheta_2} e^{\frac{y}{\vartheta_2}} \mathrm{I}_{\{y<0\}}, \tag{6}$$

where $\delta_1, \delta_2 \ge 0$, $1/\vartheta_1 > 1$, $\vartheta_2 > 0$ and $\delta_1 + \delta_2 = 1$. The variables δ_1 and δ_2 represent, respectively, the probabilities of a positive (with an average size $\vartheta_1 > 0$) and negative (with average absolute size $\vartheta_2 > 0$) jump in mortality. All sources of randomness are assumed to be independent. To price longevity derivatives, the stochastic differential equation (5) must be rewritten under the pricing measure. Regarding the diffusive component of the longevity risk, we assume that $dW_t^{\mathbb{Q}} = dW_t^P + \frac{\lambda}{\sigma}\sqrt{\mu_{x+t}(t)}$, with λ denoting a market price of longevity risk parameter. Following [15], assume that the survival probability $S(x,t,T)$ is represented by an exponentially affine function, i.e.,

$$S(x,t,T) = e^{A(\tau)+B(\tau)\mu_{x+t}(t)}, \tag{7}$$

with $\tau = T - t$. It can be shown that $A(\tau)$ and $B(\tau)$ are solutions to the following system of ODEs

$$\dot{B}(\tau) = aB(\tau) + \frac{1}{2}\sigma^2 B^2(\tau) - 1, \tag{8}$$

$$\dot{A}(\tau) = \eta\left(\frac{\delta_1}{1-\vartheta_1 B(\tau)} + \frac{\delta_2}{1+\vartheta_2 B(\tau)} - 1\right), \tag{9}$$

with boundary conditions $A(0) = 0$ and $B(0) = 0$. By solving (8)-(9), we get the following closed-form solutions for $A(\tau)$ and $B(\tau)$ and for the survival probability (7).

$$A(\tau) = \eta\delta_1\left\{\frac{\alpha_0\tau}{(\alpha_0 - \vartheta_1)} + \frac{\vartheta_1(\alpha_0+\alpha_1)[ln(\alpha_0+\alpha_1) - ln(\alpha_0 - \vartheta_1 + (\alpha_1+\vartheta_1)e^{\kappa\tau})]}{\kappa(\alpha_0-\vartheta_1)(\alpha_1+\vartheta_1)}\right\}$$
$$+\eta\delta_2\left\{\frac{\alpha_0\tau}{(\alpha_0+\vartheta_2)} + \frac{\upsilon_2(\alpha_0+\alpha_1)}{\kappa(\alpha_1-\vartheta_2)(\alpha_0+\vartheta_2)}[-ln(\alpha_0+\alpha_1)\right. \tag{10}$$
$$\left. +ln(\alpha_0+\upsilon_2+(\alpha_1-\vartheta_2)e^{\kappa\tau})]\right\} - \eta\tau$$

$$B(\tau) = \frac{1-e^{\kappa\tau}}{\alpha_0+\alpha_1 e^{\kappa\tau}}, \tag{11}$$

with $\kappa = \sqrt{a^2+2\sigma^2}$, $\alpha_0 = \frac{(a+\kappa)}{2}$ and $\alpha_1 = \frac{(\kappa-a)}{2}$, defined for $-\frac{1}{\vartheta_2} < B(\tau) < \frac{1}{\vartheta_1}$. For the financial component of the contract, given the long-term nature of survivor bonds, the HJM [33] model structure fitting the observed yield curve should be used.

2.3 Model calibration

To calibrate the model to empirical data, we follow a cohort approach and use U.S. total population mortality data obtained from the Human Mortality Database [34] for ages in the range 65-100. We consider cohorts completing 65 years from 1950 to 2017. For the discretized stochastic process, we assume that the age-specific forces of mortality are constant within yearly bands of time and age, i.e., within each square of the Lexis diagram. Under this assumption, we obtain empirical survival curves for representative cohorts using $\hat{S}(x,t,T) = exp\left(-\sum_{j=0}^{T-1} m_{x+j}(t+j)\right)$, where $m_x(t)$ is the central death rate for an individual aged x at time t. Table 1 exhibits the estimated model parameters for the illustrative cohort aged 65 in 1950, with $\mu_{65}(0) = -ln(S(65,0))$.

Table 1: Estimated model parameters, cohort aged 65 in 1950, U.S. Total Population

Parameter	a	σ	η	ϑ_1	ϑ_2	δ_1	$\mu_{65}(0)$
Estimate	0.07540775	0.00974780	0.09983138	0.00100002	0.00082434	0.00010003	0.02883801

Source: author's preparation. Note: Model's SSE=0.000208508.

The parameter estimates show that the value of the diffusion coefficient σ is very low, a result also found in similar studies. The average (absolute) size of negative mortality jumps has been declining for younger generations, potentially signaling a slowdown in longevity improvements. Figure 1 plots the observed (blue dots) and fitted (magenta line) survival probability of a U.S individual aged 65 in 1950. We can observe that the affine jump-diffusion model specified above fits very well the U.S. 65-year-old mortality dynamics.

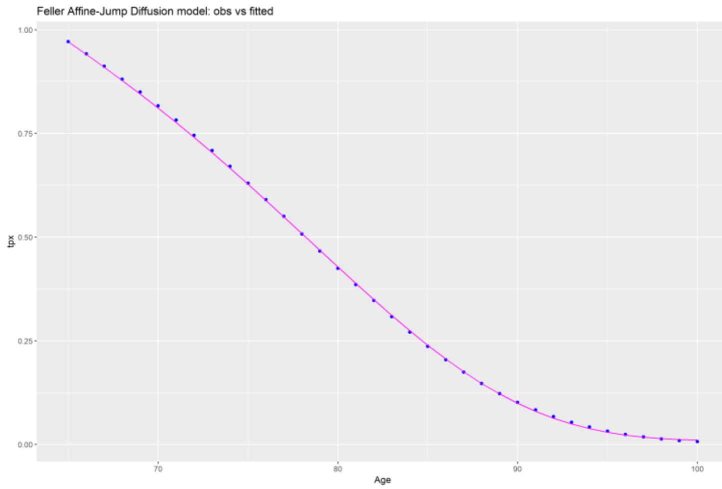

Figure 1: Affine jump-diffusion model – observed versus fitted survival probabilities.

3 RESULTS

This section reports a summary of the empirical pricing results obtained in this study. Without loss of generality, the baseline scenario is of a coupon-at-risk bond with a standard coupon $\theta = 2\%$, a flat yield curve at 2%, and the market price of longevity risk of 17%. Later we provide sensitivity analysis results for relaxing some of the key model parameters. Figure 2 plots the survivor bond relative additive margin for different maturities from one up to 50 years. As expected, the relative additive margin is largely an increasing function of the bond's maturity meaning the longer the maturity of the contract the higher the risk premium required by the investor to hold the asset. The relative additive margin values range between nearly 0.05% and nearly 2.4%. Yet, similar to Denuit et al. (2007) our results show that the relative additive margin declines slowly for very long maturity bonds (maturity higher than 32 years), suggesting for those maturities the small number of remaining survivors at very old ages and the present value effect slightly reduce the compensation demanded by investors. Our estimates for the relative additive margin suggest that the cost of issuance of such a product would be acceptable for the issuer, e.g., a pension plan or an annuity provider.

Figure 3 reports the sensitivity analysis of the survivor bond relative additive margin estimates for a 30-year contract to changes in the affine-jump diffusion stochastic mortality model key parameters. The top left panel shows the sensitivity of π^* to changes in the volatility coefficient in the range 0%-10%. The top right panel shows the sensitivity of π^* to changes in the market price of longevity risk coefficient in the range 0%-42%. The bottom left panel shows the sensitivity of π^* to changes in the jump intensity coefficient in the range 0-0.004. Finally, the bottom right panel shows the sensitivity of π^* to changes in the positive and negative jump size coefficients. Our pricing results show that the compensation required by the investor to buy the coupon-at-risk survivor bond (the relative additive margin) increases with the volatility of the underlying reference population mortality rates, increases in a linear way with the market price of longevity risk premium, increases with the intensity of jumps in the dynamics of mortality rates at old ages, it is a positive function of the average size of negative jumps in the mortality intensity (e.g., due to medical or drug breakthroughs that reduce mortality) and declines with the average size of negative jumps in the mortality rates, e.g., due to military conflicts or a pandemic.

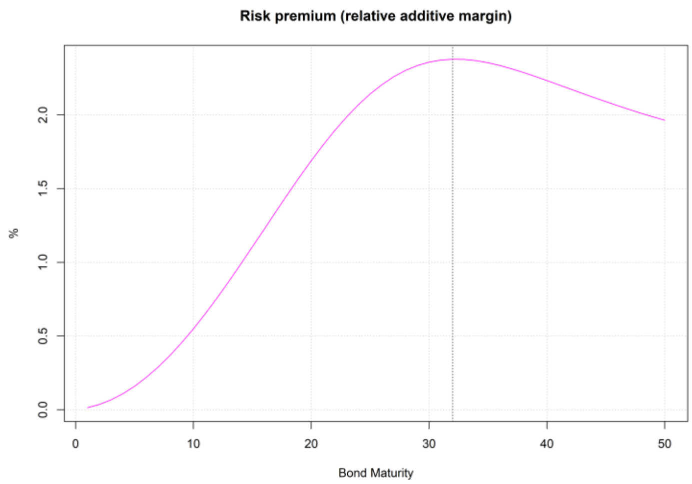

Figure 2: Survivor Bond relative additive margin estimates for different maturities

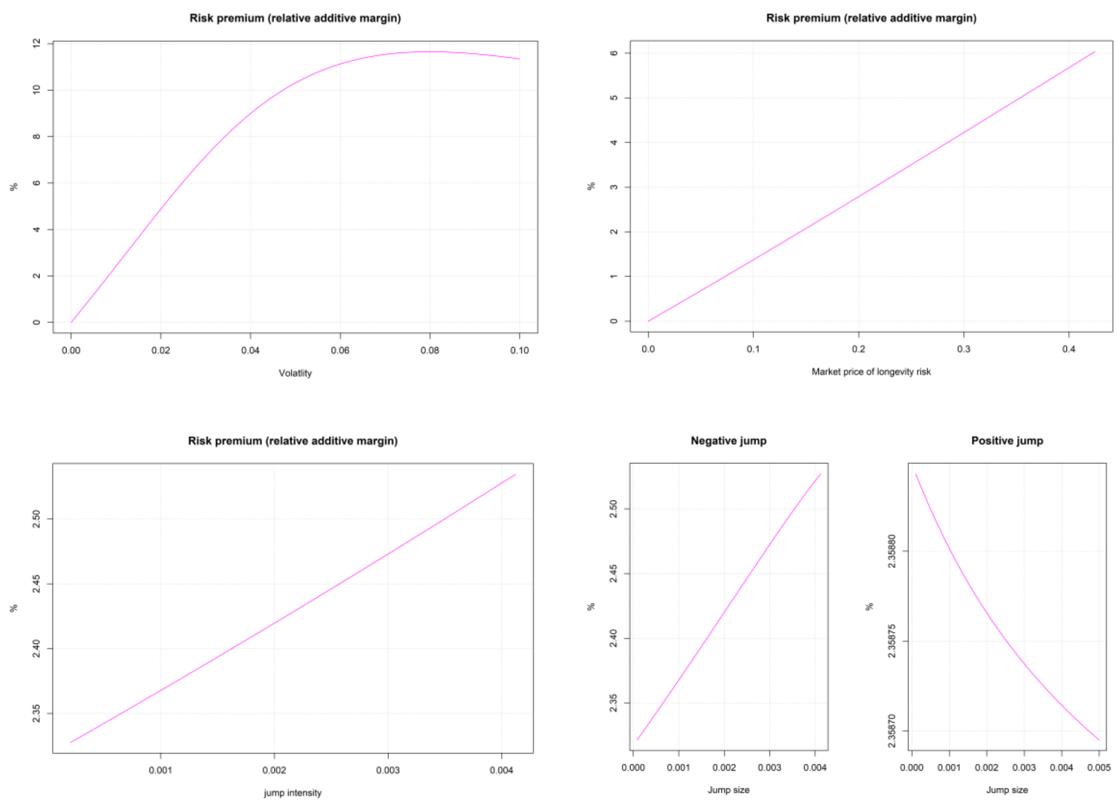

Figure 3: Sensitivity analysis of the Survivor Bond relative additive margin estimates for a 30-year contract

4 CONCLUSION

This paper uses an affine-jump diffusion framework to model mortality intensities to derive closed-form solutions to the survival probability and price coupon-at-risk survivor bonds of different maturity. The framework accounts for both negative and positive jumps of different sizes, providing a broader assessment of the uncertainty underlying the mortality rates of different populations. For pension funds and annuity providers, survivor bonds are an interesting alternative to classical insurance-based solutions for hedging longevity risk, and an interesting asset class for investors seeking to diversify their portfolios and to generate an extra return on their portfolios. The empirical results show that the cost of issuing this hedging instrument would be acceptable for the issuer, particularly when compared to expensive reinsurance solutions. Further research should investigate the inclusion of survivor bonds in ALM immunization strategies [35, 41] and account for counterparty default risk. Further research should also investigate alternative survivor bond designs with both coupon and principal linked to the survival index allowing for caps and floors. Further research should investigate the sensitivity of results to changes in the method used to incorporate the market price of longevity risk (e.g., Wang Transform distortion approach, indifference pricing principles, CCAPM, standard deviation premium principle).

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